Some Effects Caused by Solid Particles in Flows

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Theme

THE equations of conservation of angular momentum of the two phases of a particulate flow are derived, and the stress tensor determined. It was found to consist of antisymmetric elements in addition to the pressure and symmetric shear stresses in nonparticulate flows. The particulate flow properties are determined numerically throughout the flowfield from frozen to equilibrium regimes.

Contents

A study of the behavior of particulate flows near the solid boundaries is necessary to determine the effect of the presence of solid particles on the skin friction and heat transfer. In the present analysis, the particles are continuously distributed throughout the flowfield and each of the two phases has its own mean properties. The particles translational velocity is generally different in magnitude and direction from that of the gas. They also have a rotational velocity which is not equal to the gas rotational speed. The particles motion, however, is basically due to the gas motion.

If the rate of change of the angular momentum of the suspension in the volume V is set equal to the total moment of the surface forces about an axis in the k-direction passing through a point 0 within this volume we obtain

$$\int (1-\chi)\rho\varepsilon_{kij} r_i (dv_j/dt)\, dV + \int \chi\rho_p \varepsilon_{kij} r_i (dv_{p_j}/dt)\, dV + \int \chi\rho_p r_J^2 (d\omega_k/dt)\, dV = \int \varepsilon_{kij}[\partial(r_i\sigma_{lj})/\partial r_l]\, dV \quad (1)$$

where ρ and ρ_p are the fluid and solid particle material densities, χ is the volume occupied by the solid particles per unit volume of mixture, r_J is the radius of gyration of a solid particle, v and v_p are the gas and particle velocities, ω is the solid particle rotational velocity, and r is the position vector, whereas the subscripts i, j, k refer to components in these directions.

If the volume, V, is reduced to zero such that the configuration, which is made up of the boundary of the volume and the fixed point 0, retains the same shape, the last term on the left-hand side and the first term on the right-hand side of Eq. (1), approach zero in the same order as V, while the rest of the terms approach zero as $V^{4/3}$. Thus

$$\chi\rho_p r_J^2 (d\omega_k/dt) = \varepsilon_{kij}\sigma_{ij} \quad (2)$$

The stress tensor is written below as the sum of three terms. The first term is spherically symmetric and represents uniform compression, the second is a symmetrical tensor, and the third is antisymmetric

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij} + \tau_{ij}^a \quad (3)$$

where p is the pressure and

$$\tau_{ij} = 2\mu\left[\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right] \quad (4)$$

If the expression for the stress tensor given by Eq. (3) is substituted into Eq. (2), the contribution of the spherically symmetric and symmetrical tensors to the right-hand side of Eq. (2) is zero and, hence, the following relation is obtained:

$$\tau_{ij}^a = (\chi\rho_p/2) r_J^2 (d\omega_k/dt); \qquad i \neq j \neq k \quad (5)$$

Thus, the stress tensor in suspension flows consists of an antisymmetric shear stress, (τ_{ij}^a), which is equal to half the rate of change of the local angular momentum of the solid particles per unit volume, in addition to the familiar pressure and symmetric shear stress in nonparticulate fluid flows.

Some numerically determined, incompressible particulate flow properties due to the impulsive motion of an infinite flat plate[1] are presented. The effect of the initial particle concentration, χ_o and the particle to gas density ratio, ρ_p/ρ, on the flow properties was investigated. The friction coefficient is plotted against time for different particle to gas density ratios in Fig. 1. The nondimensional skin-friction and antisymmetric stress tensor are defined as

$$C_f^* = C_f (Re\, t^*)^{1/2}$$

$$C_{xy}^{a*} = \frac{2\tau_{xy}^a (Re\, t^*)^{1/2}}{\rho U^2 \chi_o}$$

The Reynolds number, Re, is based on the plate velocity, U, and the relaxation time of particle translation, τ_t

$$Re = U^2\tau_t/\nu$$

The time is normalized with respect to τ_t

$$t^* = t/\tau_t$$

where

$$\tau_t = (d^2/18\nu)(\rho_p/\rho)$$

In the last equation, d is the solid particle diameter and ν is the kinematic viscosity.

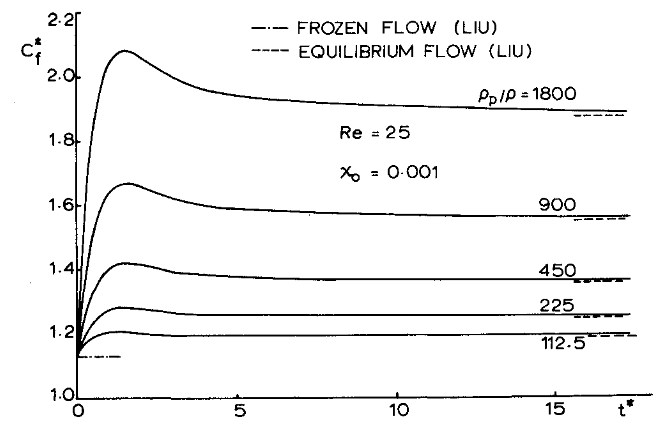

Fig. 1 Effect of particle to gas density ratio on the friction coefficient at the wall.

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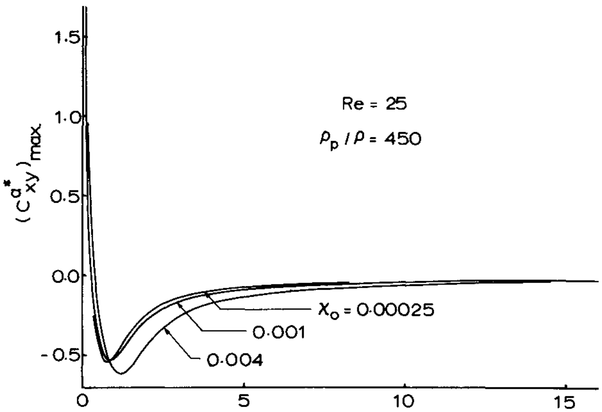

Fig. 2 Effect of the particle volume concentration on the maximum antisymmetric stress.

The friction coefficient was found to increase with increased particle concentration and increased density ratios. The antisymmetric stress tensor due to particle rotational acceleration does not contribute to the skin friction at the plate since a demixed particle region develops next to it.[1] The values of the friction coefficients obtained by Liu for frozen and equilibrium flows are shown on the same figure for comparison.[2] The variation of the maximum value of the antisymmetric shear stress with time is shown in Fig. 2. The antisymmetric shear stress reaches its maximum value next to the demixed region. It changes sign because of initial acceleration, then later deceleration, of particle rotation. A measure of the extent of the layer moving with the plate in the transformed plane are η_t and η_{t_p} for the gas and the particles, respectively, which were defined in a manner similar to the displacement thickness in nonparticulate boundary layers

$$\eta_t = \frac{1}{1-\chi_o}\int_o^\infty (1-\chi)\frac{u}{U}\,d\eta$$

$$\eta_{t_p} = \frac{1}{\chi_o}\int_{\eta_d}^\infty \chi\frac{u_p}{U}\,d\eta$$

where the coordinate normal to the plate, y, is normalized as follows:

$$\eta = [y/2(\nu t)^{1/2}]$$

and η_d is the normalized lateral distance traveled by a particle that was located initially at the plate.[3]

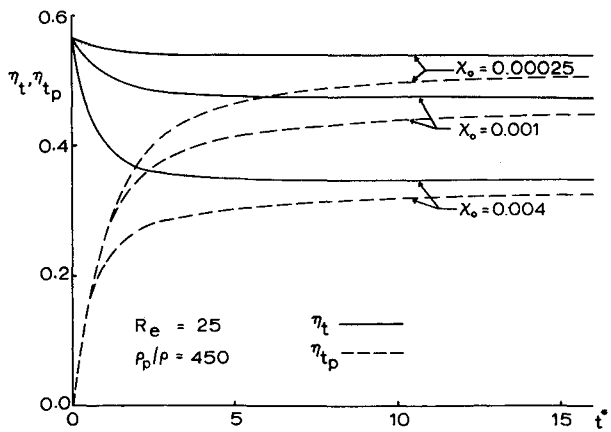

Fig. 3 Effect of particle concentration on the gas and particle displacement thickness.

Figure 3 shows that both η_t and η_{t_p} decrease with increased particle concentration. It was also found that larger density ratios produce the same effect on both displacement thicknesses as increased particle concentration. The gas displacement thickness in the transformed plane, η_t, decreases from its initial frozen value to a smaller equilibrium value resulting from the thinner gas velocity profiles due to the energy consumed in accelerating the particles. The frozen value of η_{t_p}, however, is zero and increases with time, eventually approaching the values of η_t.

References

[1] Hamed, A. and Tabakoff, W., "Analysis of Nonequilibrium Particulate Flow," AIAA Paper 73-687, Palm Springs, Calif., 1973.

[2] Liu, J. T. C., "Flow Induced by the Impulsive Motion of an Infinite Flat Plate in a Dusty Gas," *Astronautica Acta*, Vol. 13, No. 4, July–Aug. 1967, pp. 369–377.

[3] Hamed, A. and Tabakoff, W., "Solid Particle Demixing in a Suspension Flow of Viscous Gas," *Fluid Mechanics of Mixing*, edited by E. Uram and V. Goldschmidt, ASME, New York, 1973, pp. 101–115.